

Fast PageRank Approximations on Graph Engines Ioannis Mitliagkas Michael Borokhovich Alex Dimakis Constantine Caramanis

## Web Ranking

Given web graph Find "important" pages



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Given web graph Find "important" pages Rank Based on In-degree Classic Approach



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# Page Importance Described by distribution $\pi$



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Important pages are pointed to by

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### Robust

to manipulation by spammer networks











E

- Start: Gallon of water distributed evenly
- Every Iteration Each vertex spreads water evenly to successors Redistribute evenly a fraction,  $p_T = 0.15$ , of all water

Repeat until convergence

Power Iteration employed usually

#### Frog walks randomly on graph Next vertex chosen uniformly at random



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PageRank VectorMany frogs, estimate vector  $\pi$ 



## PageRank Approximation

Looking for k "heavy nodes"

Do not need full PageRank vector

#### Random Walk Sampling

Favors heavy nodes

#### Captured Mass Metric For node set S: $\pi(S)$



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Platform

- \* Engine splits graph across cluster
- \* Vertex program describes logic

#### GAS abstraction



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1. Gather



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- 2. Apply



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- 1. Gather
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- 3. Scatter



## Edge Cuts

- Assign vertices to machines
- Cross-machine edges require network communication
- Pregel, GraphLab 1.0
- High-degree nodes generate
   large volume of traffic
- Computational load imbalance



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## Vertex Cuts

- Assign edges to machines
- High-degree nodes replicated
- \* One replica designated master
- Need for synchronization
  - 1. Gather
  - 2. Apply [on master]
  - 3. Synchronize mirrors
  - 4. Scatter
- \* GraphLab 2.0 PowerGraph
- Balanced Network still bottleneck



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## Random Walks on GraphLab

Master node decides step

Decision synced to all mirrors

Only machine M needs it

Unnecessary network traffic

Average replication factor ~8



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Objective

#### Faster PageRank approximation on GraphLab

Idea Only synchronize the mirror that will receive the frog Doable, but requires

- 1. Serious engine hacking
- 2. Exposing an ugly/complicated API to programmer

SimplerPick mirrors to synchronize at random!Synchronize independently with probability  $p_S$ 



Release N frogs in parallel

### Vertex Program

1.Each frog dies w.p.  $p_T$  (gives sample) Assume K frogs survive

2.For every mirror, draw bridge w.p. $p_S$ 

3.Spread frogs evenly among synchronized mirrors.



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Bridges introduce dependencies!



### Contributions

- Algorithm for approximate PageRank
   Modification of GraphLab
   Exposes very simple API extension (p<sub>S</sub>).
   Allows for randomized synchronization.
- 3.Speedup of 7-10x
- 4. Theoretical guarantees for solution despite introduced dependencies

### Theoretical Guarantee

Mass Captured by top-k set, S, of estimate from N frogs after t steps

$$\pi(S) \ge \operatorname{OPT} - 2\epsilon \quad \text{w.p. } 1 - \delta$$
here  $\epsilon < \sqrt{k}\lambda_2^t + \sqrt{\frac{k}{\delta} \left[\frac{1}{N} + (1 - p_S^2)p_{\cap}(t)\right]}$ 

probability two Frogs meet at first t steps

W

$$p_{\cap}(t) \le \frac{1}{n} + \frac{t \|\pi\|_{\infty}}{p_T},$$



### **Experimental Results**



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### References

Page, L., Brin, S., Motwani, R., & Winograd, T. (1999). The PageRank citation ranking: Bringing order to the web.

Malewicz, Grzegorz, et al. "Pregel: a system for large-scale graph processing." Proceedings of the 2010 ACM SIGMOD International Conference on Management of data. ACM, 2010.

Low, Y., Gonzalez, J., Kyrola, A., Bickson, D., Guestrin, C., & Hellerstein, J. M. (2010). Graphlab: A new framework for parallel machine learning. arXiv preprint arXiv:1006.4990.

Gonzalez, J. E., Low, Y., Gu, H., Bickson, D., & Guestrin, C. (2012, October). PowerGraph: Distributed Graph-Parallel Computation on Natural Graphs. In OSDI (Vol. 12, No. 1, p. 2).

Nguyen, D., Lenharth, A., & Pingali, K. (2013, November). A lightweight infrastructure for graph analytics. In Proceedings of the Twenty-Fourth ACM Symposium on Operating Systems Principles (pp. 456-471). ACM.

Avery, C. (2011). Giraph: Large-scale graph processing infrastruction on Hadoop. Proceedings of Hadoop Summit. Santa Clara, USA:[sn].

Xin, R. S., Gonzalez, J. E., Franklin, M. J., & Stoica, I. (2013, June). Graphx: A resilient distributed graph system on spark. In First International Workshop on Graph Data Management Experiences and Systems (p. 2). ACM.

Backup Slides



## Normalized Adjacency Matrix $P_{ij} = \frac{1}{d_{out}(j)}, \quad (j,i) \in G$



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**Power Method**  $Q^t p^0 \to \pi$ 



Here be dragons.



## Backup

